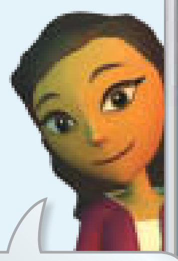
# 9-1 Translations

**Common Core State Standards**
**G-CO.A.2** Represent transformations in the plane . . . describe transformations as functions that take points in the plane as inputs and give other points as outputs . . . **Also G-CO.A.4, G-CO.B.6**
**MP 1, MP 3, MP 4, MP 7**

**Objectives** To identify rigid motions
To find translation images of figures

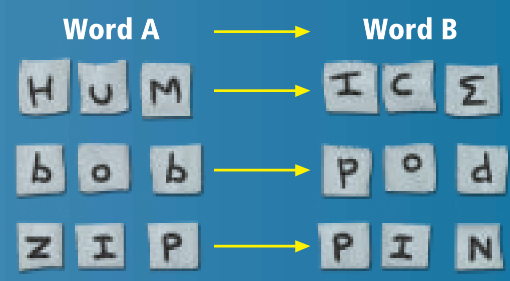


There is more than one way to move each letter. Look for the most efficient way.

MATHEMATICAL PRACTICES

**SOLVE IT!** **Getting Ready!**

Suppose you write the letters shown on squares of tracing paper so their shapes are visible from both sides. For each pair of words, how can you move the squares of paper to change Word A into Word B? Note: No square should remain in its original position.



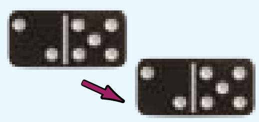

**Lesson Vocabulary**
- transformation
- preimage
- image
- rigid motion
- translation
- composition of transformations

In the Solve It, you described changes in positions of letters. In this lesson, you will learn some of the mathematical language used to describe changes in positions of geometric figures.

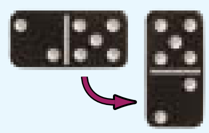
**Essential Understanding** You can change the position of a geometric figure so that the angle measures and the distance between any two points of a figure stay the same.

A **transformation** of a geometric figure is a function, or *mapping* that results in a change in the position, shape, or size of the figure. When you play dominoes, you often move the dominoes by flipping them, sliding them, or turning them. Each move is a type of transformation. The diagrams below illustrate some basic transformations that you will study.

The domino flips.



The domino slides.



The domino turns.

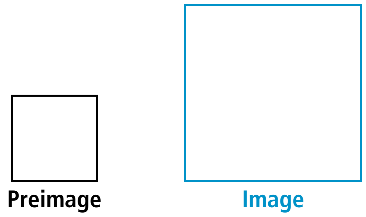
In a transformation, the original figure is the **preimage.** The resulting figure is the **image.** Some transformations, like those shown by the dominoes, preserve distance and angle measures. To preserve distance means that the distance between any two points of the image is the same as the distance between the corresponding points of the preimage. To preserve angles means that the angles of the image have the same angle measure as the corresponding angles of the preimage. A transformation that preserves distance and angle measures is called a **rigid motion**.

**Think**

**What must be true about a rigid motion?**
In a rigid motion, the image and the preimage must preserve distance and angle measures.

## Problem 1 Identifying a Rigid Motion

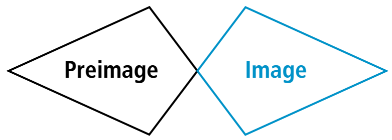
**Does the transformation at the right appear to be a rigid motion? Explain.**



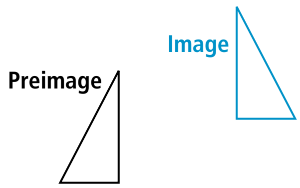

No, a rigid motion preserves both distance and angle measure. In this transformation, the distances between the vertices of the image are not the same as the corresponding distances in the preimage.

**Got It?** **1.** Does the transformation appear to be a rigid motion? Explain.

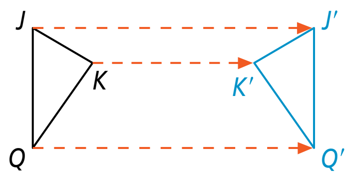
**a.**




**b.**




A transformation maps every point of a figure onto its image and may be described with arrow notation ($\rightarrow$). Prime notation ($'$) is sometimes used to identify image points. In the diagram below, $K'$ is the image of $K$.




$\triangle JKQ \rightarrow \triangle J'K'Q'$
$\triangle JKQ$ maps onto $\triangle J'K'Q'$.

Notice that you list corresponding points of the preimage and image in the same order, as you do for corresponding points of congruent or similar figures.

**Plan**

**How do you identify corresponding points?**
Corresponding points have the same position in the names of the preimage and image. You can use the statement $EFGH \rightarrow E'F'G'H'$.

## Problem 2 Naming Images and Corresponding Parts

**In the diagram, $EFGH \rightarrow E'F'G'H'$.**

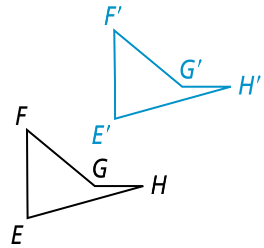
**A What are the images of $\angle F$ and $\angle H$?**

$\angle F'$ is the image of $\angle F$. $\angle H'$ is the image of $\angle H$.

**B What are the pairs of corresponding sides?**

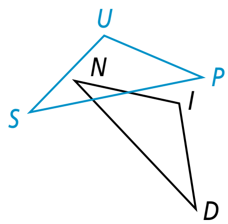
$\overline{EF}$ and $\overline{E'F'}$ $\quad$ $\overline{FG}$ and $\overline{F'G'}$

$\overline{EH}$ and $\overline{E'H'}$ $\quad$ $\overline{GH}$ and $\overline{G'H'}$



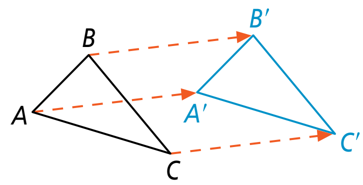

$EFGH \rightarrow E'F'G'H'$

**Got It?** **2.** In the diagram, $\triangle NID \rightarrow \triangle SUP$.
**a.** What are the images of $\angle I$ and point $D$?
**b.** What are the pairs of corresponding sides?



## take note **Key Concept** Translation

A **translation** is a transformation that maps all points of a figure the same distance in the same direction.

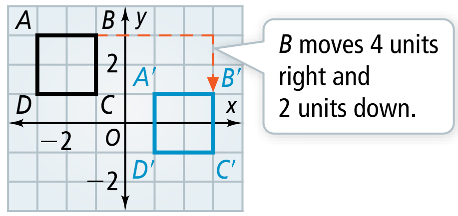



You write the translation that maps $\triangle ABC$ onto $\triangle A'B'C'$ as $T(\triangle ABC) = \triangle A'B'C'$. A translation is a rigid motion with the following properties.

If $T(\triangle ABC) = \triangle A'B'C'$, then

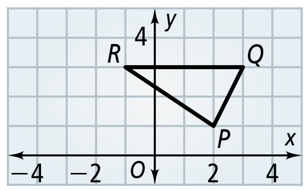
- $AA' = BB' = CC'$
- $AB = A'B'$, $BC = B'C'$, $AC = A'C'$
- $m\angle A = m\angle A'$, $m\angle B = m\angle B'$, $m\angle C = m\angle C'$

The diagram at the right shows a translation in the coordinate plane. Each point of $ABCD$ is translated 4 units right and 2 units down. So each $(x, y)$ pair in $ABCD$ is mapped to $(x + 4, y - 2)$. You can use the function notation $T_{<4,\,-2>}(ABCD) = A'B'C'D'$ to describe this translation, where 4 represents the translation of each point of the figure along the $x$-axis and $-2$ represents the translation along the $y$-axis.




## Problem 3 Finding the Image of a Translation

**What are the vertices of $T_{<-2,\,-5>}(\triangle PQR)$? Graph the image of $\triangle PQR$.**




**Think**

**What does the rule tell you about the direction each point moves?**

$-2$ means that each point moves 2 units left. $-5$ means that each point moves 5 units down.

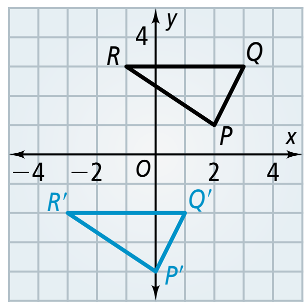
Identify the coordinates of each vertex. Use the translation rule to find the coordinates of each vertex of the image.

$T_{<-2,\,-5>}(P) = (2 - 2, 1 - 5)$, or $P'(0, -4)$.

$T_{<-2,\,-5>}(Q) = (3 - 2, 3 - 5)$, or $Q'(1, -2)$.

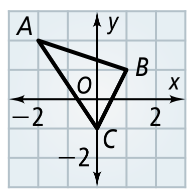
$T_{<-2,\,-5>}(R) = (-1 - 2, 3 - 5)$, or $R'(-3, -2)$.

To graph the image of $\triangle PQR$, first graph $P'$, $Q'$, and $R'$. Then draw $\overline{P'Q'}$, $\overline{Q'R'}$, and $\overline{R'P'}$.



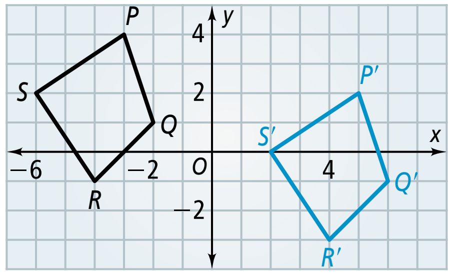

**Got It?** **3. a.** What are the vertices of $T_{<1,\,-4>}(\triangle ABC)$? Copy $\triangle ABC$ and graph its image.

**b. Reasoning** Draw $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. What relationships exist among these three segments? How do you know?



## Problem 4 Writing a Rule to Describe a Translation

**What is a rule that describes the translation that maps *PQRS* onto *P′Q′R′S′*?**




**Know**

The coordinates of the vertices of both figures

**Need**

An algebraic relationship that maps each point of *PQRS* onto *P′Q′R′S′*

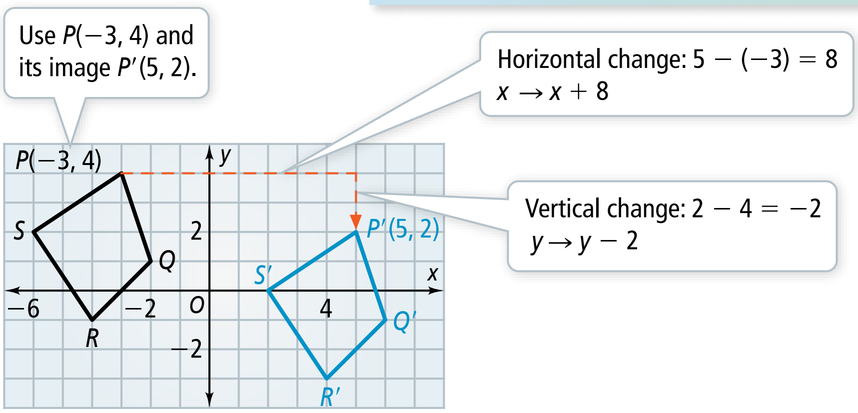
**Plan**

Use one pair of corresponding vertices to find the change in the horizontal direction *x* and the change in the vertical direction *y*. Then use the other vertices to verify.

**Think**

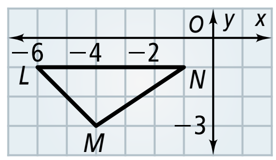
**How do you know which pair of corresponding vertices to use?**

A translation moves all points the same distance and the same direction. You can use any pair of corresponding vertices.



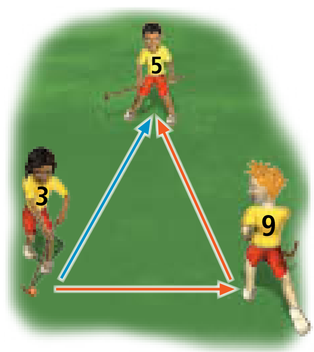

The translation maps each $(x, y)$ to $(x + 8, y - 2)$. The translation rule is $T_{<8,\,-2>}(PQRS)$.

**Got It? 4.** The translation image of $\triangle LMN$ is $\triangle L'M'N'$ with $L'(1, -2)$, $M'(3, -4)$, and $N'(6, -2)$. What is a rule that describes the translation?




A **composition of transformations** is a combination of two or more transformations. In a composition, you perform each transformation on the image of the preceding transformation.

In the diagram at the right, the field hockey ball can move from Player 3 to Player 5 by a direct pass. This translation is represented by the blue arrow. The ball can also be passed from Player 3 to Player 9, and then from Player 9 to Player 5. The two red arrows represent this composition of translations.




In general, the composition of any two translations is another translation.

## Problem 5 Composing Translations

**Chess** **The diagram at the right shows two moves of the black bishop in a chess game. Where is the bishop in relation to its original position?**


> **Think**
> **How can you define the bishop's original position?**
> You can think of the chessboard as a coordinate plane with the bishop's original position at the origin.

Use (0, 0) to represent the bishop's original position. Write translation rules to represent each move.

$T_{<4,\,-4>}(x, y) = (x + 4, y - 4)$ The bishop moves 4 squares right and 4 squares down.

$T_{<2,\,2>}(x, y) = (x + 2, y + 2)$ The bishop moves 2 squares right and 2 squares up.

The bishop's current position is the composition of the two translations.

First, $T_{<4,\,-4>}(0, 0) = (0 + 4, 0 - 4)$, or $(4, -4)$.

Then, $T_{<2,\,2>}(4, -4) = (4 + 2, -4 + 2)$, or $(6, -2)$.

The bishop is 6 squares right and 2 squares down from its original position.

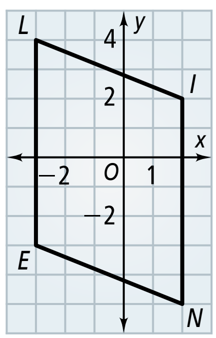
**Got It?** **5.** The bishop next moves 3 squares left and 3 squares down. Where is the bishop in relation to its original position?

## Lesson Check

### Do you know HOW?

**1.** If $\triangle JPT \rightarrow \triangle J'P'T'$, what are the images of $P$ and $\overline{TJ}$?

**2.** Copy the graph at the right. Graph $T_{<-3,\,-4>}(NILE)$.

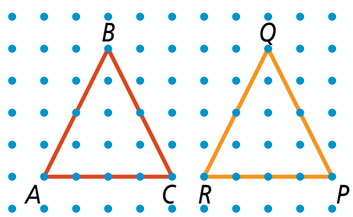



**3.** Point $H(x, y)$ moves 12 units left and 4 units up. What is a rule that describes this translation?

### Do you UNDERSTAND? MATHEMATICAL PRACTICES

**4. Vocabulary** What is true about a transformation that is not a rigid motion? Include a sketch of an example.

**5. Error Analysis** Your friend says the transformation $\triangle ABC \rightarrow \triangle PQR$ is a translation. Explain and correct her error.




**6. Reasoning** Write the translation $T_{<1,\,-3>}(x, y)$ as a composition of a horizontal translation and a vertical translation.

## Practice and Problem-Solving Exercises

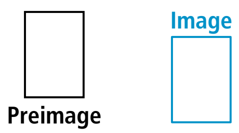
MATHEMATICAL PRACTICES

### A Practice

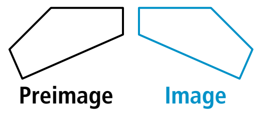
Tell whether the transformation appears to be a rigid motion. Explain.

See Problem 1.

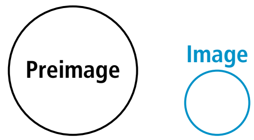
**7.**




**8.**



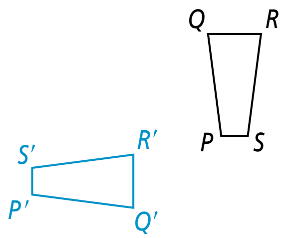

**9.**



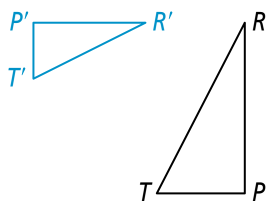

In each diagram, the blue figure is an image of the black figure.
(a) Choose an angle or point from the preimage and name its image.
(b) List all pairs of corresponding sides.

See Problem 2.

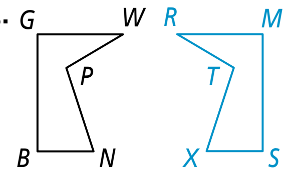
**10.**




**11.**



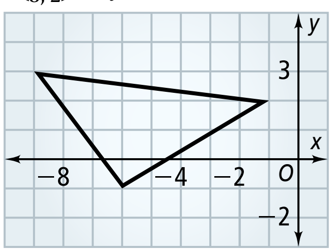

**12.**



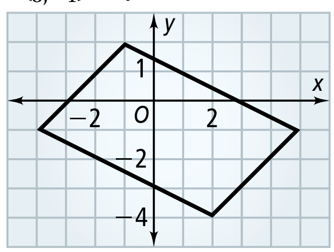

Copy each graph. Graph the image of each figure under the given translation.

See Problem 3.

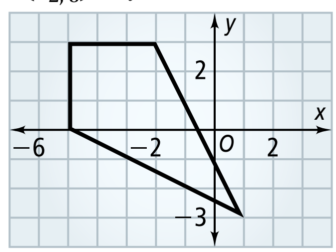
**13.** $T_{<3,\,2>}(x, y)$




**14.** $T_{<5,\,-1>}(x, y)$



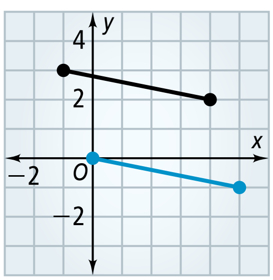

**15.** $T_{<-2,\,5>}(x, y)$



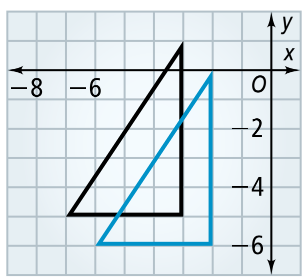

The blue figure is a translation image of the black figure. Write a rule to describe each translation.

See Problem 4.

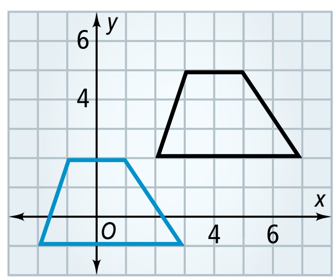
**16.**




**17.**




**18.**



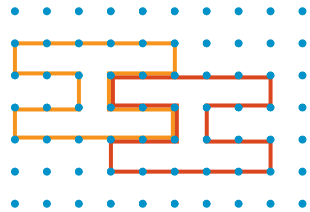

**19. Travel** You are visiting San Francisco. From your hotel near Union Square, you walk 4 blocks east and 4 blocks north to the Wells Fargo History Museum. Then you walk 5 blocks west and 3 blocks north to the Cable Car Barn Museum. Where is the Cable Car Barn Museum in relation to your hotel?

See Problem 5.

**20.** **Travel** Your friend and her parents are visiting colleges. They leave their home in Enid, Oklahoma, and drive to Tulsa, which is 107 mi east and 18 mi south of Enid. From Tulsa, they go to Norman, 83 mi west and 63 mi south of Tulsa. Where is Norman in relation to Enid?

**B Apply**

**21.** In the diagram at the right, the orange figure is a translation image of the red figure. Write a rule that describes the translation.



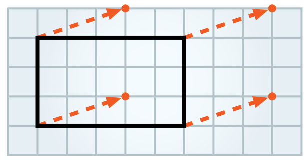
**22.** **Think About a Plan** $\triangle MUG$ has coordinates $M(2, -4)$, $U(6, 6)$, and $G(7, 2)$. A translation maps point $M$ to $M'(-3, 6)$. What are the coordinates of $U'$ and $G'$ for this translation?
- How can you use a graph to help you visualize the problem?
- How can you find a rule that describes the translation?

**23.** **Coordinate Geometry** $PLAT$ has vertices $P(-2, 0)$, $L(-1, 1)$, $A(0, 1)$, and $T(-1, 0)$. The translation $T_{<2, -3>}(PLAT) = P'L'A'T'$. Show that $\overline{PP'}$, $\overline{LL'}$, $\overline{AA'}$, and $\overline{TT'}$ are all parallel.

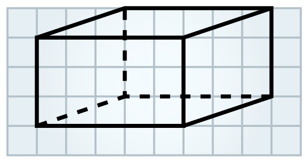
**Geometry in 3 Dimensions** **Follow the sample at the right. Use each figure, graph paper, and the given translation to draw a three-dimensional figure.**

**SAMPLE** Use the rectangle and the translation $T_{<3, 1>}(x, y)$ to draw a box.

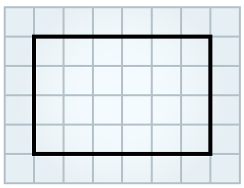
**Step 1**



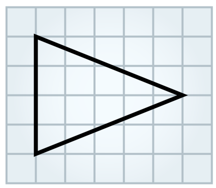
**Step 2**



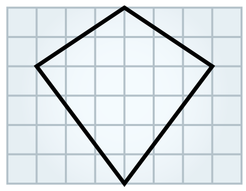
**24.** $T_{<2, -1>}(x, y)$



**25.** $T_{<-2, 2>}(x, y)$



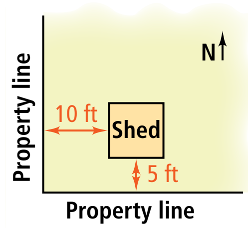
**26.** $T_{<-3, -5>}(x, y)$



**27.** **Open-Ended** You are a graphic designer for a company that manufactures wrapping paper. Make a design for wrapping paper that involves translations.

**28.** **Reasoning** If $T_{<5, 7>}(\triangle MNO) = \triangle M'N'O'$, what translation rule maps $\triangle M'N'O'$ onto $\triangle MNO$?

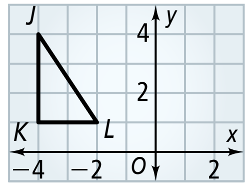
**29.** **Landscaping** The diagram at the right shows the site plan for a backyard storage shed. Local law, however, requires the shed to sit at least 15 ft from property lines. Describe how to move the shed to comply with the law.




STEM **30.** **Computer Animation** You write a computer animation program to help young children learn the alphabet. The program draws a letter, erases the letter, and makes it reappear in a new location two times. The program uses the following composition of translations to move the letter.

$T_{<5, 7>}(x, y)$ followed by $T_{<-9, -2>}(x, y)$

Suppose the program makes the letter W by connecting the points (1, 2), (2, 0), (3, 2), (4, 0) and (5, 2). What points does the program connect to make the last W?




**31.** Use the graph at the right. Write three different translation rules for which the image of $\triangle JKL$ has a vertex at the origin.

**Find a translation that has the same effect as each composition of translations.**

**32.** $T_{<2,\,5>}(x, y)$ followed by $T_{<-4,\,9>}(x, y)$

**33.** $T_{<12,\,0.5>}(x, y)$ followed by $T_{<1,\,-3>}(x, y)$

**Challenge**

**34. Coordinate Geometry** $\triangle ABC$ has vertices $A(-2, 5)$, $B(-4, -1)$, and $C(2, -3)$. If $T_{<4,\,2>}(\triangle ABC) = \triangle A'B'C'$, show that the images of the midpoints of the sides of $\triangle ABC$ are the midpoints of the sides of $\triangle A'B'C'$.

**35. Writing** Explain how to use translations to draw a parallelogram.

## Standardized Test Prep

**SAT/ACT**

**36.** $\triangle ABC$ has vertices $A(-5, 2)$, $B(0, -4)$, and $C(3, 3)$. What are the vertices of the image of $\triangle ABC$ after the translation $T_{<7,\,-5>}(\triangle ABC)$?

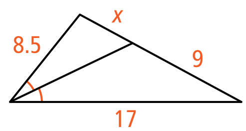
(A) $A'(2, -3)$, $B'(7, -9)$, $C'(10, -2)$

(B) $A'(-12, -3)$, $B'(-7, -9)$, $C'(-4, -2)$

(C) $A'(-12, 7)$, $B'(-7, 1)$, $C'(-4, 8)$

(D) $A'(2, -3)$, $B'(10, -2)$, $C'(7, -9)$

**37.** What is the value of $x$ in the figure at the right?




(F) 4.5

(G) 16

(H) 18

(I) 18.5

**38.** In $\triangle PQR$, $PQ = 4.5$, $QR = 4.4$, and $RP = 4.6$. Which statement is true?

(A) $m\angle P + m\angle Q < m\angle R$

(B) $\angle Q$ is the largest angle.

(C) $\angle R$ is the largest angle.

(D) $m\angle R < m\angle P$

**Short Response**

**39.** $\square ABCD$ has vertices $A(0, -3)$, $B(-4, -2)$, and $D(-1, 1)$.

**a.** What are the coordinates of $C$?

**b.** Is $\square ABCD$ a rhombus? Explain.

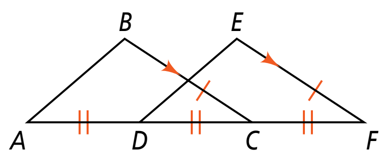
## Mixed Review

**40. Navigation** An airplane landed at a point 100 km east and 420 km south from where it took off. If the airplane flew in a straight line from where it took off to where it lands, how far did it fly?

See Lesson 8-1.

**41. Given:** $\overline{BC} \cong \overline{EF}$, $\overline{BC} \parallel \overline{EF}$, $\overline{AD} \cong \overline{DC} \cong \overline{CF}$

**Prove:** $\overline{AB} \cong \overline{DE}$




See Lesson 4-7.

**Get Ready!** **To prepare for Lesson 9-2, do Exercises 42–44.**

**Write an equation for the line through $A$ perpendicular to the given line.**

See Lesson 3-8.

**42.** $A(1, -2)$; $x = -2$

**43.** $A(-1, -1)$; $y = 1$

**44.** $A(-1, 2)$; $y = x$